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Technical Notes

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Retarded-Time Calculation for Moving Sources

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Nomenclature

A = moving surface
c = speed of sound
M_f = flight Mach number
M_r = Mach number of source in radiation direction
M_t = tip rotation Mach number
p = acoustic pressure
Q = acoustic source
R = $|\boldsymbol{r}|$
$\boldsymbol{r}$ = $\boldsymbol{x}(t) - \boldsymbol{y}(\tau)$
t = observer time
$\boldsymbol{x}(t)$ = observer position
$\boldsymbol{y}(\tau)$ = source position
ϵ = error tolerance
τ = retarded time
Ω = rotation frequency, rad/s

Introduction

IN a recent paper on the application of integral methods to the prediction of aerodynamically generated noise,[1] Brentner discusses the problem of evaluating the retarded time for a moving source. A retarded-time method, of which a representative example is Farassat's formulation,[2] contains an integral of the form

$$4\pi p(\boldsymbol{x}, t) = \int_A \left[\frac{Q(\boldsymbol{y}, \tau)}{R|1 - M_r|}\right] \mathrm{d}A \tag{1}$$

where [ ] denotes evaluation of the enclosed quantity at the retarded time τ. The standard approach for calculating the radiated noise is to fix t and calculate τ at the quadrature points on the moving surface A, integrating to find the radiated noise. This evaluation must be performed for each point in the pressure time record and for each quadrature point on the moving surface. Because the motion of the surface can be extremely complex (for example, see the helicopter noise problems considered in Ref. 3), the retarded time cannot be calculated analytically and must be found using a numerical root-finding technique. This is time consuming, and alternative approaches have been considered. Brentner[1] discusses the source-time dominant approach in which τ is treated as the independent variable. If τ is specified, t can usually be calculated analytically because the observer is typically stationary or in rectilinear motion. To calculate the noise radiated by a point on the surface, τ is discretized and the corresponding values of t calculated. These values of t will be unevenly spaced, however, and will not be the same for different points on the surface. An extra step is needed to interpolate the unevenly spaced pressures onto a set of evenly spaced time points in order to superpose them and calculate the overall radiated noise.

This Note proposes an alternative retarded-time evaluation method, which uses a source-time dominant approach to generate an interpolation table for the retarded time, with a specified error bound.

Retarded-Time Algorithm

In evaluating the retarded time τ, the equation to be solved is

$$\tau = t - R(\tau)/c \tag{2}$$

where the distance R is a function of source and observer positions. It is assumed that a reference frame is chosen in which the observer is stationary or in rectilinear motion.

For any specified value of τ, we can calculate, exactly, t. A simple method of calculating the retarded time would be to generate a list of points (t_i, τ_i) and interpolate in this list to calculate τ for any value of t. Using this approach, however, we would not know in advance how long a list to generate for a specified accuracy nor would we know the error in the interpolated values.

An alternative approach is as follows: for a given value of τ, calculate t and derivatives $\mathrm{d}^n\tau/\mathrm{d}t^n$. The derivatives can be used for interpolation and to estimate the error in the evaluated retarded time. Near some point (t_i, τ_i)

$$\tau = \sum_{n=0}^{N} \frac{1}{n!} \left.\frac{\mathrm{d}^n\tau}{\mathrm{d}t^n}\right|_{t=t_i} (\Delta t)^n + \mathcal{O}[(\Delta t)^{N+1}] \tag{3}$$

where $\Delta t = t - t_i$. Noting that $\mathrm{d}/\mathrm{d}t = (\mathrm{d}\tau/\mathrm{d}t)\mathrm{d}/\mathrm{d}\tau$, the first four derivatives can be written as

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{1 - M_r} \tag{4a}$$

$$\frac{\mathrm{d}^2\tau}{\mathrm{d}t^2} = \frac{\dot{M}_r}{(1 - M_r)^3} \tag{4b}$$

$$\frac{\mathrm{d}^3\tau}{\mathrm{d}t^3} = \frac{3\dot{M}_r^2}{(1 - M_r)^5} + \frac{\ddot{M}_r^2}{(1 - M_r)^4} \tag{4c}$$

$$\frac{\mathrm{d}^4\tau}{\mathrm{d}t^4} = \frac{15\dot{M}_r^3}{(1 - M_r)^7} + \frac{10\dot{M}_r\ddot{M}_r}{(1 - M_r)^6} + \frac{\dddot{M}_r^2}{(1 - M_r)^5} \tag{4d}$$

Using these derivatives gives enough information for third-order-accurate evaluation of τ, with the fourth-order term acting as the error estimate. All derivatives on the right-hand side are with respect to τ, that is, depend on the source motion, which is known, with the relative Mach number given by

$$M_r = -\frac{1}{c}\frac{\mathrm{d}R}{\mathrm{d}\tau} \tag{5}$$

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Finally, we need a method of differentiating R in order to calculate M_r and its derivatives, given the source motion. Noting that

$$R^2 = \boldsymbol{r} \cdot \boldsymbol{r}$$

we apply Leibnitz's rule for the derivatives of a product[4]:

$$\frac{\mathrm{d}^n}{\mathrm{d}\tau^n}(R^2) = \sum_{q=0}^{n} \binom{n}{q} \boldsymbol{r}^{(q)} \cdot \boldsymbol{r}^{(n-q)} = \sum_{q=0}^{n} \binom{n}{q} \frac{\mathrm{d}^q R}{\mathrm{d}\tau^q} \frac{\mathrm{d}^{n-q} R}{\mathrm{d}\tau^{n-q}}$$

so that, upon rearrangement,

$$\frac{\mathrm{d}^n R}{\mathrm{d}\tau^n} = \frac{1}{2R}\left[\sum_{q=0}^{n} \binom{n}{q} \boldsymbol{r}^{(q)} \cdot \boldsymbol{r}^{(n-q)} - \sum_{q=1}^{n-1} \binom{n}{q} \frac{\mathrm{d}^q R}{\mathrm{d}\tau^q} \frac{\mathrm{d}^{n-q} R}{\mathrm{d}\tau^{n-q}}\right] \tag{6}$$

To generate a retarded-time interpolation table with $\mathcal{O}[(\Delta t)^n]$ error, proceed as follows. For the initial value of $\tau = \tau_0$, calculate t_0 and the corresponding derivatives [Eq. (4)]. Calculate the next value of $\tau = \tau + \Delta\tau$ using the local expansion (3), $\Delta\tau$ being found using the error criterion

$$\Delta\tau \leq (n!\epsilon/|\mathrm{d}^n\tau/\mathrm{d}t^n|)^{1/n} \tag{7}$$

For this value of $\tau = \tau_1$, calculate t_1 and the corresponding derivatives, repeating the procedure until the required range of τ has been covered. This generates a set of points t with exact values of τ and derivatives. To calculate τ for some value of t, use the local expansion (3) about the point t_i nearest the required value t.

Results

The algorithm just described has been tested for accuracy and computational effort by implementing it as a MATLAB® routine and comparing it to the performance of MATLAB's internal root-finding function. This is an efficient technique based on the secant method (Ref. 5, pp. 159–161) and was used to generate a reference solution for the retarded times, which could be compared with that produced by the proposed algorithm. To speed the calculation of retarded time at each time point, the retarded time for the preceding time point was used as an initial guess for the root. As a measure of performance, the total number of floating point operations (flops) was used. Two problems were considered in order to demonstrate the application of the method in different situations.

The first problem, representative of in-flight propeller noise, is that of calculating retarded times for a rotating source and observer in forward flight. The source rotates at constant helical Mach number $M_h = 1$, and the flight Mach number M_f is varied from 0 to 0.866 so that the tip rotational Mach number M_t varies from 1 to 0.5. The observer is placed 2 source radii from the axis of rotation and 10^{-3} radii ahead of the plane of rotation. This means that M_r is always less than one but is almost sonic when $M_f = 0$. For comparison, retarded times were calculated to the same tolerance ϵ using the MATLAB root-finding function. As a reference for evaluating the accuracy, the root-finding approach was used to calculate the retarded time to close to machine precision ($\epsilon = 10^{-15}$). A record of length 512 points was calculated in each case.

Table 1 shows the maximum nomdimensional retarded time error for values of $\Omega\epsilon = 10^{-5}$, 10^{-4}, and 10^{-3}, corresponding to phase errors of 0.0036, 0.036, and 0.36 deg, respectively. It can be seen that the errors are all within the specified limits. Table 2 shows the ratio of the number of floating point operations required for the proposed algorithm compared to the root-finding approach. In each case the effort required is less than 10% of that using the standard method.

Table 1 Algorithm performance: maximum nondimensional error/specified tolerance

	$\Omega\epsilon$		
M_t	10^{-3}	10^{-4}	10^{-5}
1.0	0.087	0.043	0.144
0.707	0.089	0.043	0.144
0.5	0.263	0.043	0.142

Table 2 Algorithm performance: ratio of floating point operations

	$\Omega\epsilon$		
M_t	10^{-3}	10^{-4}	10^{-5}
1.0	0.027	0.045	0.073
0.707	0.027	0.044	0.072
0.5	0.026	0.044	0.071

The second case considered is that of a helicopter rotor undergoing a maneuver. A study of the noise from such a system has been published recently,[3] and we consider a simplified model of the last case in that paper, the problem of noise generated during a 3-deg arrested descent. In Ref. 3 the input to the acoustic calculation is only available at discrete retarded times because of the complex motion of the rotor. In that spirit, we compute the retarded times for the tip of a rotor of diameter 14 m, in forward flight at 62 m/s (121 kn) and rotating at 5 Hz so that the maximum tip Mach number is 0.51. The center of the rotor descends on a parabolic path with mean angle 3 deg and then makes a transition to horizontal flight. The parabolic path was chosen to provide an analytical description of the flight path for input to the standard root-finding routine and in order to avoid unphysical jumps in velocity at the point of transition to horizontal flight. Using an analytically prescribed trajectory also allowed an accurate error check to be performed.

To introduce some extra complexity similar to that caused by changes in aircraft pitch, the rotation axis was made perpendicular to the flight path at all times. The input to the retarded-time computation was the source position $\boldsymbol{y}(\tau)$ sampled every 5 deg of a rotation as in Ref. 3; no source velocities or higher-order quantities were used. The observer time for each retarded time was calculated for an observer 30 m below the center of rotation at time $t = 0$.

The retarded times were calculated at evenly spaced observer times by linear interpolation in the look-up table (t_i, τ_i) and by root finding as just described. The accuracy of the interpolated retarded times was checked against those found by root finding, and the maximum error was found to be $\Omega\epsilon = 1.3 \times 10^{-4}$. The computational effort for the proposed algorithm was 2.5% of that for root finding. If a cubic spline was used to interpolate the retarded time, the maximum error dropped to 6.3×10^{-5}, and the computational effort was 5.9% of that for root finding.

Finally, we note that when only source positions are available the error constraint from the power series cannot be used. In this case a suitable error measure for an interpolated value of τ is $\epsilon = t - \tau - R/c$, where the source position is computed (by interpolation, say) at the interpolated value of τ. This error measure was tested in this case and was found to be practically identical to the error found by comparison with the root-finding method.

Conclusions

A new method for the numerical evaluation of retarded times in acoustics has been proposed. The method is simple to implement using standard numerical procedures and requires no special root-finding techniques. It has been tested in two cases representative of modern problems in aerodynamically generated noise and has proven accurate and highly efficient in both, even when only limited data are available for the source motion. In this case the algorithm demonstrated its accuracy and robustness, even using minimal information about the complex motion of the source.

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Estimation of the Flowfield from Surface Pressure Measurements in an Open Cavity

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Introduction

IT is generally accepted that the vortical structures formed in the shear layer above the cavity are convected downstream and interact with the cavity's downstream wall. The interaction sets up a feedback loop that reinforces the development of the vortices in the shear layer causing the cavity to resonate. Although this general description of cavity flow is known, there is still a need for time-resolved information about the shear layer dynamics in order to better understand the feedback process and other noise sources in the shear layer above the cavity. Historically, the large body of work focusing on cavity flows has usually taken one of three forms: evaluation of pressure loads inside of the cavity, evaluation of phase-aligned flow dynamics using optical techniques such as schlieren or particle image velocimetry (PIV) or numerical simulation of Navier–Stokes equations. In cases where the flow dynamics were captured through visual techniques, limitations involved in recording and saving images prevented the technique from yielding time-resolved measurements and coupling them with the surface pressure dynamics. This problem is somewhat overcome in a numerical simulation where time-dependent information can be resolved; however, three-dimensional, time-dependent numerical simulations are quite large, often limiting the Reynolds number and typically requiring a large amount of computing resources. For a more detailed survey of the literature on cavity flows one should refer to many papers on the subject, such as Refs. 1–4.

Recent emphasis in resonating cavity research has focused on both active[5–7] and passive[8–11] control. Passive control strategies have resulted in a favorable reduction in pressure loads for design conditions, but they are unable to adapt to off-design conditions and often exhibit adverse effects in those conditions. Therefore, active control is necessary to yield an adaptive control system to address dynamic cavity flow problems. For active control to be successful there is a need for time-resolved information about flowfield dynamics to be implemented with an adaptive control strategy for optimization in a range of flow conditions. Current experimental techniques are limited in their ability to yield time-resolved whole-field measurements. The current effort focuses on producing a low-dimensional, time-dependent description of the flow to be utilized in producing the necessary information for developing an adaptive control scheme.

As a precursor to a large experimental effort to study the dynamics of the shear layer above an open cavity, the present investigation focuses on the evaluation of stochastic estimation for its use in estimating velocities in the shear layer using multiple surface pressure measurements as predictors. Utilizing surface pressure measurements in the stochastic estimation of velocity is a relatively new idea and allows for applications in many practical situations where difficulties arise from placing probes in the flow. Additionally, using a stochastic estimation technique will allow for constructing time-resolved flowfield details that cannot be readily obtained from current experimental techniques. A multipoint method was recently employed for studying a backward-facing ramp by Taylor,[12] and single-point pressure estimation has been more thoroughly detailed by Naguib et al.[13] The present work utilizes the large eddy simulations of a Mach 1.5 freestream flow over a cavity, with a length-to-depth ratio (L/D) of 6, to extend these earlier formulations for multiple pressure measurements in both linear and quadratic estimates of the flow over an open cavity. Utilizing the data set from time-resolved simulations, one can directly examine the time dependence of the estimated velocity field by comparing snapshots of the flowfield in addition to the statistical properties, which are typically used, to evaluate the effectiveness of linear and nonlinear applications of stochastic estimation.

Stochastic Estimation

Stochastic estimation was presented by Adrian[14] in 1975 as a means of estimating coherent structures in turbulent flows. Later, Cole et al.[15] showed that by utilizing the instantaneous velocity as the condition the time dependence of the velocity field could be estimated. As shown by Picard and Delville,[16] Taylor,[12] and Naguib et al.,[13] this conditional average can be formulated using the wall pressure event as the condition:

$$\tilde{u}_{ijx}(t) = \langle u_{ijx}(t) | P(t) \rangle \tag{1}$$

The subscripts are used to denote the position i and j (two-dimensional flow) and the component x of the velocity that is of interest. Angle brackets denote ensemble averaging, and the velocities and pressures represent only the fluctuating components.

The conditional average can be estimated by a power series as shown by Guezennec[17]:

$$\tilde{u}_{ijx}(t) = A_{ijxk} P_k(t) + B_{ijxlm} P_l(t) P_m(t) + C_{ijxpqr} P_p(t) P_q(t) P_r(t) + \cdots \tag{2}$$

The summation convention has been utilized, and the sum is taken from 1 to K, where K is the number of estimating events (surface pressure measurements in this case). The coefficients are found by minimizing the mean square error of the estimate depending on where the estimate is truncated.

To evaluate the success of the estimate, both the linear stochastic estimate (LSE) and quadratic stochastic estimate (QSE) expansions were formulated for multiple predictors and evaluated for their performance. The formulation of the multipoint estimates for the LSE using either velocity or pressure can be found in many places, and so we will detail only the formulation for the quadratic estimation. The quadratic estimate involves the first two terms of Eq. (2):

$$\tilde{u}_{ijx}(t) = A'_{ijxk} P_k(t) + B_{ijxlm} P_l(t) P_m(t) \tag{3}$$

Note the prime above the linear coefficients in Eq. (3). Owing to the calculation of the coefficients, the linear and quadratic coefficients are not independent, and so the coefficients for the linear term in the quadratic estimate are slightly different from the coefficients in the linear estimate.

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